Exercise 1:

I tried writing a python script to find what would be a counterexample for the problem, but it just ran forever so I am just going to assume that there is no counterexample because the statement (n is an odd integer if and only if 3n+5 is an even integer) is true.

If you’re not looking for a value for n then I’d say that theoretically this could be disproven if n is odd and 3n+5 is odd.

Exercise 2:

|  |  |  |  |
| --- | --- | --- | --- |
| N | N^2 | 2^N | N^2>=2^N |
| 2 | 4 | 4 | T |
| 3 | 9 | 8 | T |
| 4 | 16 | 16 | T |

Exercise 3:

If x is even and y is odd then x+y is odd

1. X is even (hyp.)
2. Y is odd (hyp.)
3. X = 2m (definition of even)
4. Y = 2n+1 (definition of odd)
5. X+Y = 2m+2n+1 (definition of odd)

Since any even plus another even plus 1 is always odd, the sum of an even and odd is always odd.

Exercise 4:

If x^2 + 2x – 3 = 0, then x!=2

1. X = 2 (hyp.)
2. (2)^2 + 2(2) – 3 (substitution)
3. 4+4-3=5 (arithmetic)
4. 5!=0 (solution)

Exercise 5:  
2+4+6+…+2n = n(n + 1)  
  
a.   
P(1) = 2(1) = 1(1 + 1)  
P(1) = 2 = 2  
  
  
  
  
  
b.

1. P(1) is true
2. P(k)🡪P(k+1) is true

Since 1 and 2 are true, P(n) is true for all positive integers n

2+4+6+…+2k = k(k+1)

c.

P(k+1): 2+4+6+…+ 2k + 2(k+1) = (k+1)(k+2)

d.  
2 + 4 + 6 + … + 2k + 2(k+1) = k(k+1) + 2(k+1) (from inductive hypothesis)

k(k+1) + 2(k+1) which once (k+1) is factored is (k+1)(k+2) which is equal to the right side of the equation in step c.

Exercise 6:

1. 2+6+18+…+2\*(3^(n-1)) = (3^n)-1

Base Case:   
S(1) = 2 \* 3^(1-1) = (3^1)-1 both of which are equivalent to 2

Induction:   
S(k) = 2+6+18+…+2\*(3^(k-1)) = (3^k)-1

S(k+1) = 2+6+18+… + 2\*(3^(k-1)) + 2\*(3^k) = (3^(k+1))-1

Using the assumption S(k) we can get:

= ((3^k)-1) + 2 \* 3^k

= 3^k + 2 \* 3^k-1

= 3 \* 3^k – 1

= 3^(k+1) – 1

1. 1 \* (2^1) + 2 \* (2^2) + 3 \* (2^3) + … + n \* (2^n) = (n-1) \* 2^(n+1) + 2

Base Case:

T(1): 1 \* 2^1 = 2

Induction:

T(k): 1 \* (2^1) + 2 \* (2^2) + 3 \* (2^3) + … + k \* (2^k) = (k-1) \* 2^(k+1) + 2

T(k+1): 1 \* (2^1) + 2 \* (2^2) + 3 \* (2^3) + … + k \* (2^k) + (k+1) \* (2^(k+1))

T(k+1) = (k-1) \* 2^(k+1) + 2 + (k+1) \* 2^(k+1)

T(k+1) = k \* 2^(k+1) + 2^(k+1) + 2

T(k+1) = (k+1) \* 2^(k+1) + 2